Present Worth Analysis

- Present Worth (PW) analysis decision criteria
- One of three decision criteria should be used when performing present worth (PW) analysis, as follows:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Appropriate Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>All alternatives have same costs</td>
<td>Maximize PW (benefits)</td>
</tr>
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<td>All alternatives have same benefits</td>
<td>Minimize PW (costs)</td>
</tr>
<tr>
<td>Neither costs nor benefits same</td>
<td>Maximize PW (Benefits) - PW (costs)</td>
</tr>
</tbody>
</table>

**Net Present Worth (NPW) is identical to PW (benefits) - PW (costs)**
Present Worth Analysis (Example)

- Example: A printed circuit board assembly company plans to install one of two wave soldering machines in its plant.
  - The machines provide equivalent service.
  - The selection will be made by PW analysis.
  - What criterion should be used to make the selection?
- Answer: Select the machine with the lower PW (costs), because "equivalent service" means that the benefits are identical for the two machines.

PW analysis with equal lives

- This is the most straightforward case of PW analysis
  - Set the analysis period equal to the lives of the alternatives
  - Choose an appropriate decision criterion
  - Make the PW calculations and choose an alternative.
- Example: A company with a minimum attractive rate of return (MARR) of 10% plans to install one of two wave soldering machines in the assembly plant.
  - Doing nothing is not an option.
  - The machines provide equivalent service and each has an estimated 20-year service life.
  - The installed cost of machine X is $20,000 with an annual operating and maintenance (O&M) cost of $3,000.
  - The installed cost of machine Y is $25,000 with an annual operating and maintenance (O&M) cost of $2,000.
PW analysis with equal lives (continued)

• Because the machines provide equivalent service (same benefits), the appropriate PW analysis criterion is to minimize PW (costs).

X: PW (costs) = $20k + $3k (P/A,10%,20)
= $20k + $3k (8.514) = $20k + $25.542k = $45,542

Y: PW (costs) = $25k + $2k (P/A,10%,20)
= $25k + $2k (8.514) = $25k + $17.028k = $42,028

Install machine Y because it has the lower PW (costs).

PW analysis with unequal lives

• When alternatives under consideration have unequal lives, one approach is to use an analysis period that is the least common multiple of the alternative lives.
• For example, if X has a 3-year life, Y has a 4-year life, and Z has a 6-year life, then a 12-year analysis period is used.
• In such cases, we assume that each alternative can be identically replaced at the end of its service life.
PW analysis with unequal lives (continued)

• Example: A company with an MARR of 10% plans to install one of three production machines (X, Y or Z) that provide equivalent service (same benefits).
  – Doing nothing is not an option.
  – The machines have zero salvage values at the end of their lives.
  – The machines are expected to have the same annual operating and maintenance (O&M) costs, although their initial costs and service lives differ, as follows:

<table>
<thead>
<tr>
<th>Machine:</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
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<tbody>
<tr>
<td>Initial cost ($)</td>
<td>25,000</td>
<td>30,000</td>
<td>50,000</td>
</tr>
<tr>
<td>Service life (years)</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
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Select a 12-year analysis period, the least common multiple of 3, 4 and 6.

The appropriate criterion is to select the machine with the lowest PW (cost) over the 12-year analysis period, assuming that:
  – X is identically replaced at EOY 3, EOY 6 and EOY 9
  – Y is identically replaced at EOY 4 and EOY 8
  – Z is identically replaced at EOY 6
PW analysis with unequal lives (continued)

X: PW (cost) = $25k + $25k (P/F,10%,3) + $25k (P/F,10%,6) 
+ $25k (P/F,10%,9)
PW (cost) = $25k [ 1 + (P/F,10%,3) + (P/F,10%,6) + (P/F,10%,9) ]
PW (cost) = $25k [ 1 + 0.7513 + 0.5645 + 0.4241 ]
= $25k (2.7399) = $68,498

Y: PW (cost) = $30k [ 1 + (P/F,10%,4) + (P/F,10%,8) ]
PW (cost) = $30k [ 1 + 0.6830 + 0.4665 ]
PW (cost) = $30k (2.1495) = $64,485

Z: PW (cost) = $50k [ 1 + (P/F,10%,6) ]
PW (cost) = $50k [ 1 + 0.5645 ] = $50k (1.5645) = $78,225

Select machine Y because it has the lowest PW (cost).

Infinite analysis period (capitalized cost)

• Capitalized cost is defined as the present worth of costs over an infinite analysis period.
• It can be shown that the factor (P/A, i%, n = infinity) is equal to (1 / i ), with the interest rate i in decimal form.
  – For example, (P/A,5%, n = infinity) = 1 / 0.05 = 20

• Example: Determine the capitalized cost at 15% interest of a structure with an initial cost of $200,000 and annual operating and maintenance costs of $40,000.
  P = $200,000 + $40,000 (P/A,15%,n = infinity)
  = $200,000 + $40,000 (1 / 0.15 )
  P = $200,000 + $266,667 = $466,667
Bond problems

- Corporations issue bonds to raise capital for investments.
- The face value of a bond is the amount that the owner of the bond will receive when the bond matures.
- A bond typically pays interest (sometimes called dividends) to the bondholder until maturity.

Bond problems (example)

- Example: Suppose that a $10,000 (face value) bond pays 6% interest quarterly (which means 1.5% of the face value per quarter), beginning exactly one quarter from today.
- Suppose further that the bond matures exactly 10 years from now.
- If this bond is purchased today, the purchaser will receive interest (or a dividend) of $150 each quarter for the next 40 quarters, plus $10,000 40 quarters from now.
Bond problems (example)

• If the investor's MARR is 16%, compounded quarterly, what is the maximum amount that the investor should pay today for this bond?

\[
PW \text{ (benefits)} = 150 \text{ (P/A,4\%,40)} + 10,000 \text{ (P/F,4\%,40)}
\]

\[
PW \text{ (benefits)} = 150 \times (19.793) + 10,000 \times (0.2083) = 5,052
\]

• Note that the 6% interest rate is used only to calculate the quarterly interest (dividend) payment to the bondholder. The investor's MARR is used to calculate the PW of the bond.

Annual Cash Flow Analysis

• Annual cash flow analysis decision criteria
• One of three decision criteria should be used when performing annual cash flow analysis, as follows:

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<td>Neither costs nor benefits same</td>
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Annual Cash Flow Analysis (Example)

• Example: A company plans to install one of two elevators in one of its office buildings.
  – The elevators provide equivalent service.
  – The selection will be made by annual cash flow analysis.
  – What criterion should be used to make the selection?
• Answer: Select the elevator with the lower EUAC (costs), because "equivalent service" means that the benefits are identical for the two elevators.

Capital investment cost

• Notation:
  EUAC = equivalent uniform annual cost
  O&M cost = operating and maintenance cost
  P = asset initial cost
  S = asset salvage value end of life
• The capital cost of an asset is the cost to purchase and install it, and then dispose of it at the end of its life.
• A positive salvage value at the end of the asset's life is treated as a negative cost.
• Note that capital costs explicitly exclude O&M costs.
Capital investment cost (continued)

• When we write any equation for cost, a negative cash flow becomes a positive cost.
  – This is because it is the custom in the United States not to say, for example, that a new car costs $30,000.
  – We say the car costs $30,000.
• EUAC for asset capital cost = annualized cost to purchase, install and later dispose of an asset.
• Therefore,
  \[ \text{EUAC for asset capital cost} = P \left( \frac{A}{P}, i\%, n \right) - S \left( \frac{A}{F}, i\%, n \right) \]
• Using the identity \( \left( \frac{A}{F}, i\%, n \right) = \left( \frac{A}{P}, i\%, n \right) - i \) we obtain:
  \[ \text{EUAC for asset capital cost} = (P - S) \left( \frac{A}{P}, i\%, n \right) + S \times i \]

Capital investment cost (continued)

• Example: An asset with an initial cost of $12,000, including installation, has an estimated salvage value of $2,000 at the end of its estimated 5-year life.
  – Using an MARR of 15%, what is the equivalent uniform annual cost of owning this asset, not including O&M costs?
• P = $12,000, S = $2,000, n = 5 years
• If the MARR were zero, then the annualized capital cost would be \( \left( \frac{P - S}{n} \right) \) or $2,000 per year.
• With an MARR greater than zero, however, the annualized cost will be greater than $2,000 because the money tied up in this asset for five years could presumably be earning interest at the MARR rate.
  \[ \text{EUAC for asset capital cost} = (P - S) \left( \frac{A}{P}, i\%, n \right) + Si \]
  \[ = (12,000 - 2,000)\left( \frac{A}{P}, 15\%, 5 \right) + (2,000) \left( 0.15 \right) \]
  \[ = (10,000) \left( 0.2983 \right) + 300 \]
  \[ = $2,983 + $300 = $3,283 \text{ per year} \]
**Total EUAC**

- The total equivalent uniform annual cost (EUAC) of an asset is given by:
  \[ \text{EUAC} = \text{EUAC of capital costs} + \text{EUA (O&M)} \]
- Example: An asset has an initial cost of $100,000 and an estimated salvage value of $40,000 after its 6-year service life. Estimated O&M costs are $50,000 in year one, increasing by $6,000 per thereafter. Calculate the total EUAC of this asset at 20% interest.

  \[
  \text{EUAC of capital costs} = (P - S)(A/P,20\%,6) + S \cdot i \\
  = (60,000)(0.3007) + (40,000)(0.20) \\
  = $18,042 + $8,000 = $26,042 \text{ per year}
  \]

  \[
  \text{EUA (O&M)} = $50,000 + $6,000 (A/G,20\%,6) \\
  = $50,000 + $6,000 (1.979) \\
  = $50,000 + $11,874 = $61,874 \text{ per year}
  \]

  Total EUAC = $26,042 + $61,874 = $87,916 \text{ per year}

**Maximize (EUAB - EUAC) problems**

- Notation:
  \[ \text{EUAB} = \text{equivalent uniform annual benefits} \]
  \[ \text{EUAC} = \text{equivalent uniform annual costs} \]
- When evaluating a single alternative using annual cash flow analysis, the alternative is recommended for investment if (EUAB - EUAC) is positive or zero at the MARR. Otherwise, reject the investment.
- Example: An asset has an initial cost of $100,000 and an estimated salvage value of $40,000 after its 6-year service life. Estimated O&M costs are $50,000 in year one, increasing by $6,000 per thereafter. The asset is expected to generate an annual benefit of $110,000.
  - Is this a desirable investment if the MARR is 20%?
Maximize (EUAB - EUAC) problems

EUAC of capital costs = (P - S)(A/P,20%,6) + Si
= (60,000)(0.3007) + (40,000)(0.20)
= $18,042 + $8,000 = $26,042 per year

EUA (O&M) = $50,000 + $6,000 (A/G,20%,6)
= $50,000 + $6,000 (1.979)
= $50,000 + $11,874 = $61,874 per year

Total EUAC = $26,042 + $61,874 = $87,916 per year

EUAB = $110,000

EUAB - EUAC = $110,000 - $87,916
= $22,084 > 0, so yes, the investment is recommended.