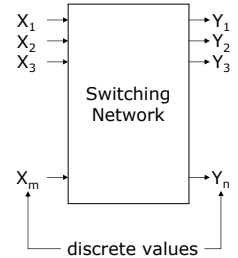


ECE380 Digital Logic

Introduction to Logic Circuits:
Variables, functions, truth tables,
gates and networks

Logic circuits

- Logic circuits perform operations on digital signals
 - Implemented as electronic circuits where signal values are restricted to a few discrete values
- In **binary** logic circuits there are only two values, 0 and 1
- The general form of a logic circuit is a switching network



Boolean algebra

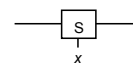
- Direct application to switching networks
 - Work with 2-state devices \rightarrow 2-valued Boolean algebra (switching algebra)
 - Use a Boolean variable (X , Y , etc.) to represent an input or output of a switching network
 - Variable may take on only two values (0, 1)
 - $X=0$, $X=1$
 - These symbols are not binary numbers, they simply represent the 2 states of a Boolean variable
 - They are not voltage levels, although they commonly refer to the low or high voltage input/output of some circuit element

Variables and functions

- The simplest binary element is a switch that has two states
- If the switch is controlled by x , we say the switch is open if $x = 0$ and closed if $x = 1$



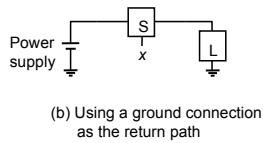
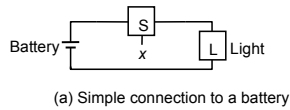
(a) Two states of a switch



(b) Symbol for a switch

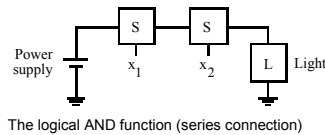
Variables and functions

- Assume the switch controls a lightbulb as shown
 - The output is defined as the state of the light L
 - If the light is on $\rightarrow L=1$
 - If the light is off $\rightarrow L=0$
- The state of L, as function of x is
 - $L(x)=x$
- $L(x)$ is a **logic function**
- x is an **input variable**



Variables and functions (AND)

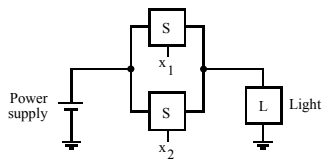
- Consider the possibility of two switches controlling the state of the light
- Using a series connection, the light will be on only if both switches are closed
 - $L(x_1, x_2) = x_1 \cdot x_2$
 - $L=1$ iff (if and only if) x_1 AND x_2 are 1



" \cdot " AND operator
 $x_1 \cdot x_2 = x_1 x_2$
 The circuit implements a logical **AND** function

Variables and functions (OR)

- Using a parallel connection, the light will be on only if either or both switches are closed
 - $L(x_1, x_2) = x_1 + x_2$
 - $L=1$ if x_1 OR x_2 is 1 (or both)

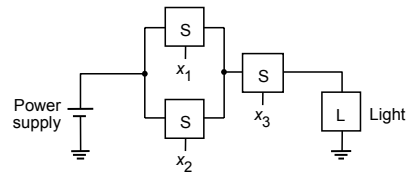


" $+$ " OR operator

The circuit implements a logical **OR** function

Variables and functions

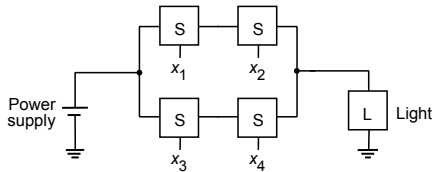
- Various series-parallel connections would realize various logic functions
 - $L(x_1, x_2, x_3) = (x_1 + x_2) \cdot x_3$



Variables and functions

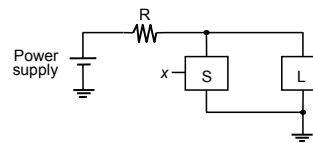
- What would the following logic function look like if implemented via switches?

$$- L(x_1, x_2, x_3, x_4) = (x_1 \cdot x_2) + (x_3 \cdot x_4)$$



Inversion

- Before, actions occur when a switch is closed. What about the possibility of an action occurring when a switch is opened?
 - $L(x) = \bar{x}$
 - Where $L=1$ if $x=0$ and $L=0$ if $x=1$
- $L(x)$ is the inverse (or complement) of x



$\bar{x}, x', \text{NOT } x$

The circuit implements a logical **NOT** function

Inversion of a function

- If a function is defined as
 - $f(x_1, x_2) = x_1 + x_2$
- Then the complement of f is
 - $\bar{f}(x_1, x_2) = \overline{x_1 + x_2} = (x_1 + x_2)'$
- Similarly, if
 - $f(x_1, x_2) = x_1 \cdot x_2$
- Then the complement of f is
 - $\bar{f}(x_1, x_2) = \overline{x_1 \cdot x_2} = (x_1 \cdot x_2)'$

Truth tables

- Tabular listing that fully describes a logic function
 - Output value for all input combinations (valuations)

x_1	x_2	$x_1 \cdot x_2$	x_1	x_2	$x_1 + x_2$	x_1	x_1'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1	NOT	
1	1	1	1	1	1		
AND			OR				

Truth tables

- Truth table for AND and OR functions of three variables

x_1	x_2	x_3	$x_1 \cdot x_2 \cdot x_3$	$x_1 + x_2 + x_3$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Truth tables of functions

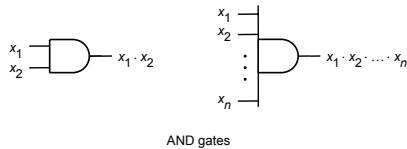
- If $L(x,y,z)=x+yz$, then the truth table for L is:

+

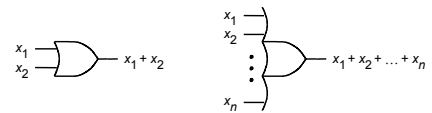
x	y	z	yz	x+yz
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Logic gates and networks

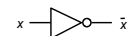
- Each basic logic operation (AND, OR, NOT) can be implemented resulting in a circuit element called a **logic gate**
- A logic gate has one or more inputs and one output that is a function of its inputs



Logic gates and networks



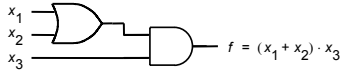
OR gates



NOT gate

Logic gates and networks

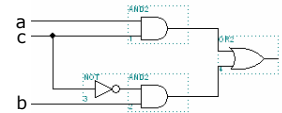
- A larger circuit is implemented by a network of gates
 - Called a logic network or logic circuit



Logic gates and networks

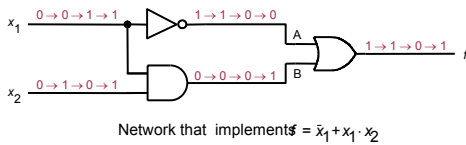
- Draw the truth table and the logic circuit for the following function
 - $F(a,b,c) = ac + bc'$

a	b	c	ac	bc'	ac+bc'
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	1	0	1



Analysis of a logic network

- To determine the functional behavior of a logic network, we can apply all possible input signals to it



Analysis of a logic network

- The function of a logic network can also be described by a timing diagram (gives dynamic behavior of the network)

