ECE380 Digital Logic

Introduction to Logic Circuits:
Boolean algebra

Axioms of Boolean algebra

- Boolean algebra: based on a set of rules derived from a small number of basic assumptions (***axioms***)

- 1a $0 \cdot 0=0$
- 1b $1+1=1$
- 2a $1 \cdot 1=1$
- 2b $0+0=0$
- 3a $0 \cdot 1=1 \cdot 0=0$
- 3b $1+0=0+1=1$
- 4a If $x=0$ then $x'=1$
- 4b If $x=1$ then $x'=0$

Single-Variable theorems

- From the axioms are derived some rules for dealing with single variables

- 5a $x \cdot 0=0$
- *5b* $x+1=1$
- *6a* $x \cdot 1=x$
- *6b* $x+0=x$
- *7a* $x \cdot x=x$
- *7b* $x+x=x$
- *8a* $x \cdot x'=0$
- *8b* $x+x'=1$
- *9* $x''=x$

- Single-variable theorems can be proven by perfect induction
- Substitute the values x=0 and x=1 into the expressions and verify using the basic axioms

Duality

- Axioms and single-variable theorems are expressed in pairs
  - Reflects the importance of ***duality***
- Given any logic expression, its dual is formed by replacing all + with ·, and vice versa and replacing all 0s with 1s and vice versa

  - f(a,b)=a+b   dual of f(a,b)=a·b
  - f(x)=x+0   dual of f(x)=x·1

- The dual of any true statement is also true

Two & three variable properties

- 10a. $x \cdot y=y \cdot x$ *Commutative*
- 10b. $x+y=y+x$

- 11a. $x \cdot (y \cdot z)=(x \cdot y) \cdot z$ *Associative*
- 11b. $x+(y+z)=(x+y)+z$

- 12a. $x \cdot (y+z)=x \cdot y+x \cdot z$ *Distributive*
- 12b. $x+y \cdot z=(x+y) \cdot (x+z)$

- 13a. $x+x \cdot y=x$ *Absorption*
- 13b. $x \cdot (x+y)=x$

Two & three variable properties

- 14a. $x \cdot y+x \cdot y'=x$ *Combining*
- 14b. $(x+y) \cdot (x+y')=x$

- 15a. $(x \cdot y)'=x'+y'$ *DeMorgan's Theorem*
- 15b. $(x+y)'=x' \cdot y'$

- 16a. $x+x' \cdot y=x+y$
- 16b. $x \cdot (x'+y)=x \cdot y$

Induction proof of $x+x' \cdot y=x+y$

- Use perfect induction to prove $x+x' \cdot y=x+y$

x	y	$x'y$	$x+x'y$	$x+y$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equivalent

Perfect induction example

- Use perfect induction to prove $(xy)'=x'+y'$

x	y	xy	$(xy)'$	x'	y'	$x'+y'$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

equivalent

Proof (algebraic manipulation)

- Prove
  - (X+A)(X'+A)(A+C)(A+D)X = AX
  - (X+A)(X'+A)(A+C)(A+D)X
  - (X+A)(X'+A)(A+CD)X (using *12b*)
  - (X+A)(X'+A)(A+CD)X
  - (A)(A+CD)X (using *14b*)
  - (A)(A+CD)X
  - AX (using *13b*)

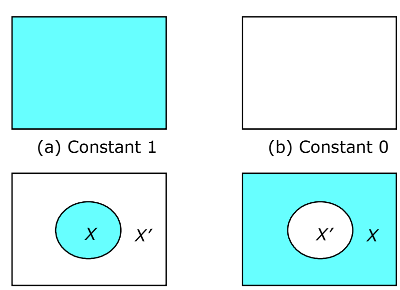
Algebraic manipulation

- Algebraic manipulation can be used to simplify Boolean expressions
  - Simpler expression => simpler logic circuit
- Not practical to deal with complex expressions in this way
- However, the theorems & properties provide the basis for automating the synthesis of logic circuits in CAD tools
  - To understand the CAD tools the designer should be aware of the fundamental concepts

Venn diagrams

- Venn diagram: graphical illustration of various operations and relations in an algebra of sets
- A set *s* is a collection of elements that are members of *s* (for us this would be a collection of Boolean variables and/or constants)
- Elements of the set are represented by the area enclosed by a contour (usually a circle)

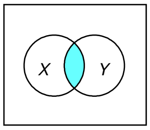
Venn diagrams



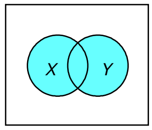
(a) Constant 1 (b) Constant 0

(c) Variable *X* (d) *X'*

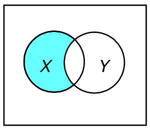
Venn diagrams



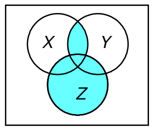
(e) *XY*



(f) *X+Y*

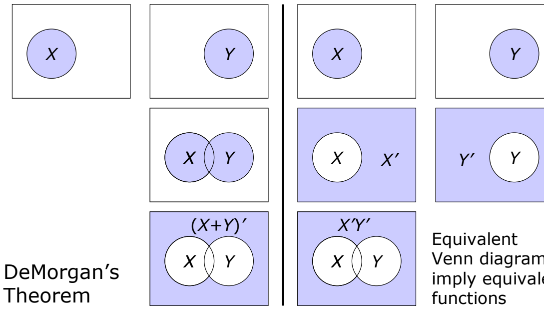


(g) *XY′*



(h) *XY+Z*

Venn diagrams (*x+y*)′= *x′y′*



DeMorgan's Theorem

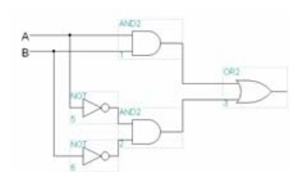
Equivalent Venn diagrams imply equivalent functions

Notation and terminology

- Because of the similarity with arithmetic addition and multiplication operations, the **OR** and **AND** operations are often called the ***logical sum*** and ***product*** operations
- The expression
  - ABC+A'BD+ACE'
  - Is a sum of three product terms
- The expression
  - (A+B+C)(A'+B+D)(A+C+E')
  - Is a product of three sum terms

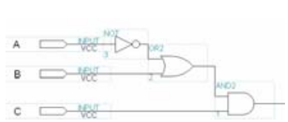
Precedence of operations

- In the absence of parentheses, operations in a logical expression are performed in the order
  - NOT, AND, OR
- Thus in the expression AB+A'B', the variables in the second term are complemented before being ANDed together. That term is then ORed with the ANDed combination of A and B (the AB term)



Precedence of operations

- Draw the circuit diagrams for the following
  - f(a,b,c)=(a'+b)c



  - f(a,b,c)=a'b+c