

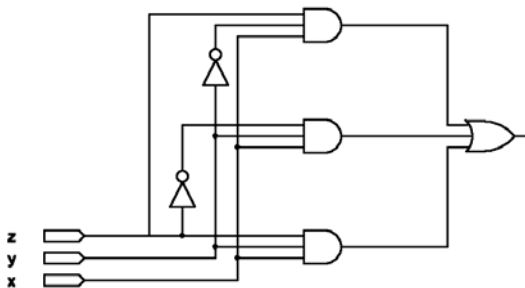
ECE380 Digital Logic

Introduction to Logic Circuits:
Synthesis using AND, OR, and NOT gates

Example logic circuit design

- Assume we want to design a logic circuit with three inputs x , y , and z
- The circuit output should be 1 only when $x=1$ and either y or z (or both) is 1
 - Three possible combinations
 - $x=1, y=0, z=1 \Rightarrow xy'z$
 - $x=1, y=1, z=0 \Rightarrow xyz'$
 - $x=1, y=1, z=1 \Rightarrow xyz$
- The function could be written as
 - $f(x,y,z)=xy'z+xyz'+xyz$
 - sum of products form

Example logic circuit design



Example logic circuit design

- Implements f correctly, $f(x,y,z)=xy'z+xyz'+xyz$ but is not the simplest such network

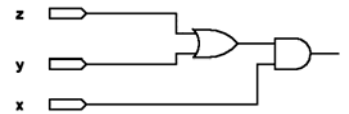
x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$xy'z+xyz'+xyz$$

$$xy'z+xy \quad \text{from 14a}$$

$$x(y'z+y) \quad \text{from 12a}$$

$$x(y+z) \quad \text{from 16a}$$



Example logic circuit design

- Obviously, the cost (in terms of gates and connections) of this network is much less than the initial network
- The process of generating a circuit from a stated desired functional behavior is called **synthesis**
- Generation of AND-OR style networks from a truth table is one of many types of synthesis techniques that we will cover

Logic synthesis

- If a function f is described in a truth table, then an expression that generates f can be obtained (synthesized) by
 - Considering all rows in the table where $f=1$, or
 - By considering all rows in the table where $f=0$
- This will be an application of the principal of duality

Minterms

- For a function of n variables $f(a,b,c,\dots,n)$
 - A minterm of f is a product of n literals (variables) in which each variable appears once in either true or complemented form, but not both
 - $f(a,b,c)$ -- minterm examples: abc , $a'bc$, abc'
 - $f(a,b,c)$ -- invalid examples: ab , c' , $a'c$
 - An n variable function has 2^n valid minterms

Minterms

Row number	x	y	z	Minterm
0	0	0	0	$m_0 = x'y'z'$
1	0	0	1	$m_1 = x'y'z$
2	0	1	0	$m_2 = x'yz'$
3	0	1	1	$m_3 = x'yz$
4	1	0	0	$m_4 = xy'z'$
5	1	0	1	$m_5 = xy'z$
6	1	1	0	$m_6 = xyz'$
7	1	1	1	$m_7 = xyz$

- Each row of a truth row corresponds to a single minterm
- When a function is written as a sum of minterms, the form is called a standard (or canonical) **sum-of-products**

Minterm notation

- An equation may be written in terms of m-notation

$$f(a,b,c) = m_0 + m_1 + m_2 + m_4$$

$$f(a,b,c) = a'b'c' + a'b'c + a'bc' + ab'c'$$

$$\begin{array}{cccc} \underbrace{000} & \underbrace{001} & \underbrace{010} & \underbrace{100} \\ 0 & 1 & 2 & 4 \end{array}$$

$$f(a,b,c) = \sum m(0,1,2,4)$$

a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Minterm notation examples

- What is the minterm notation for the following function?
 - $f(a,b,c) = abc + a'bc + abc' + a'b'c$
- What is the function (in terms of variables) if the minterm notation is the following?
 - $f(a,b,c) = \sum m(1,5,6,7)$

Logic synthesis

- Duality suggests that:
 - If it is possible to synthesize a function f by considering the truth table rows where $f=1$, then it should also be possible to synthesize f by considering the rows for which $f=0$.
- This approach uses the complement of minterms, which are called **maxterms**

Maxterms

Row number	x	y	z	Maxterm
0	0	0	0	$M_0 = x+y+z$
1	0	0	1	$M_1 = x+y+z'$
2	0	1	0	$M_2 = x+y'+z$
3	0	1	1	$M_3 = x+y'+z'$
4	1	0	0	$M_4 = x'+y+z$
5	1	0	1	$M_5 = x'+y+z'$
6	1	1	0	$M_6 = x'+y'+z$
7	1	1	1	$M_7 = x'+y'+z'$

- Each row of a truth row corresponds to a single maxterm
- When a function is written as a product of maxterms, the form is called a standard (or canonical) **product-of-sums**

Maxterm notation

- An equation may be written in terms of M-notation

$$f(a,b,c) = M_3 \cdot M_5 \cdot M_6 \cdot M_7$$

$$f(a,b,c) = (a+b'+c')(a'+b+c')(a'+b'+c)(a'+b'+c')$$

$$\underbrace{0\ 1\ 1}_3 \quad \underbrace{1\ 0\ 1}_5 \quad \underbrace{1\ 1\ 0}_6 \quad \underbrace{1\ 1\ 1}_7$$

$$f(a,b,c) = \Pi M(3,5,6,7)$$

a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Maxterm notation examples

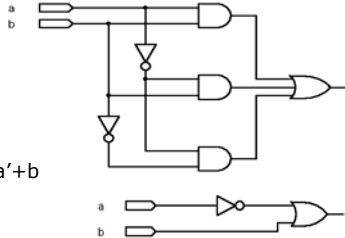
- What is the maxterm notation for the following function?
 - $f(a,b,c) = (a+b+c)(a'+b+c)(a+b+c')(a'+b'+c)$
- What is the function (in terms of variables) if the maxterm notation is the following?
 - $f(a,b,c) = \Pi M(1,5,6,7)$

Sum-of-products and minimality

- A function expressed in standard sum-of-products (or product-of-sums) form may not be minimal

a	b	f
0	0	1
0	1	1
1	0	0
1	1	1

$$f(a,b) = a'b' + a'b + ab = a' + b$$



Form conversion

- If a function f is given in Σm or ΠM form, it is easy to find f or f' in Σm or ΠM form
- Use the following form conversion table

GIVEN FORM	DESIRED FORM			
	$f = \Sigma m$	$f = \Pi M$	$f' = \Sigma m$	$f' = \Pi M$
$f = \Sigma m$ (0,2,5,7)	--	Use numbers <u>not</u> on minterm list (1,3,4,6)	Use numbers <u>not</u> on minterm list (1,3,4,6)	Use numbers on minterm list (0,2,5,7)
$f = \Pi M$ (1,3,4,6)	Use numbers <u>not</u> on maxterm list (0,2,5,7)	--	Use numbers on maxterm list (1,3,4,6)	Use numbers <u>not</u> on maxterm list (0,2,5,7)